**STAT 6106: Applied Bayesian Methods**

### Mid-term Examination

### Oct. 12th, 2018 8:00pm-9:30pm (90min)

**Policies**:

•This is an open-book exam. You may use any printed/written materials and a calculator.

•All work must be your own. You must not give or receive aid of any kind.

•This handout includes 1-page questions (3 problems).

•Only your writing on the answer book will be graded.

**Problem 1 (10 points).** Describe what is Bayesian inference and its difference from Frequentist’s approach.

**Problem 2 (40 points).** Let’s consider the customer queue in front of an ATM. We assume the inter-arrival times between two consecutive arrivals (denoted as ***X***) follows an Exponential distribution, i.e., . Daniel wants to use Bayesian method to estimate the parameter , which is the mean value of ***X.***

**(a) (8 points)** What type of distribution is the conjugate prior distribution for ? Give the distribution name and its general density function.

**(b). (4 points)** Daniel was told that the average inter-arrival time is around 10minutes. How to choose a specific distribution as your prior from the conjugate distribution family in **(a)**?

**(c). (10 points)** Daniel observed the queue in front of this ATM for a whole afternoon. In total, he saw 20 customers, and the time from the first customer’s arrival to the last customer’s arrival is 5 hours. Please derive the posterior distribution.

**(d). (8 points)** Please give both a point estimate and an interval estimate of by summarizing your posterior distribution. If you cannot give the exact number, please describe the method or the logic.

**(e). (10 points)** What is your prediction of the arrival time of the next new customer?

**Problem 3 (50 points).** ***Yoshinori Ohsumi*** from Japan won the 2016 Nobel Prize in Physiology/Medicine “for his discoveries of mechanisms for ***autophagy***” (a biological process in cells, meaning “self eating”). To show the existence of the ***autophagy*** phenomenon in cells, ***Ohsumi*** did a groundbreaking experiment. Basically, if a cell shows no ***autophagy*** phenomenon, he shall observe which follows . But if a cell shows the ***autophagy*** phenomenon, he shall observe which follows .

**(a) (10 points)** Assume and equal prior probability. Assume he observed y=9 from a cell, what is the posterior probability that the cell showed no ***autophagy***.

**(b) (10 points)** If we are sure that a cell shows ***autophagy*** and we know the mean value of is 5. Please perform Bayesian inference for . (Note: you need to justify your prior selection and derive the corresponding posterior).

**(c) (20 points)** Assume . If we observed *yi* from *n* i.i.d cells, please estimate the probability that a cell shows ***autophagy.***

**(d) (10 points)** Now assume he observed y1=6 from . and y2=7 from independently, what is the probability that